Probability of agreement

Mário de Castro[1]

[1]USP/ICMC, São Carlos, SP, Brazil
mcastro@icmc.usp.br

Regional Program MATH-AmSud

July 28, 2020

Background

1. Correlation coefficients have a long history: Pearson, Spearman, Kendall, ...
2. A new coefficient of correlation
   Chatterjee, S. (2020), *Journal of the American Statistical Association*, https://doi.org/10.1080/10.1080/01621459.2020.1758115.
3. The Hellinger correlation
   Geenens, G. & de Micheaux, P. L. (2020), *Journal of the American Statistical Association*, https://doi.org/10.1080/01621459.2020.1791132.
4. Concordance correlation coefficient (*CCC*)
   Lin. L. I. (1989). A concordance correlation coefficient to evaluate reproducibility. *Biometrics, 45, 255–268.*
   Lin, L. , Hedayat, A. S., Sinha, B., & Yang, M. (2002). Statistical methods in assessing agreement: models, issues, and tools. *Journal of the American Statistical Association* 97, 257–270.

Background

Pandit, V. & Schuller, B. (2020). The many-to-many mapping between the concordance correlation coefficient, and the mean square error. https://arxiv.org/abs/1902.05180:

$MSE_1 < MSE_2$ does not imply $CCC_1 > CCC_2$ (counterintuitive).

Probability of agreement

1. Stevens, N. T., Steiner, S. H. , & MacKay, R. J. (2017). Assessing agreement between two measurement systems: An alternative to the limits of agreement approach. *Statistical Methods in Medical Research* 26, 2487–2504.

   The probability of agreement is related to the coverage probability (Lin *et al.*, 2002).

   Model in Chan, L. K. & Mak, T. K. (1979). Likelihood estimation of a linear structural relationship with replication. *Journal of the Royal Statistical Society B* 41, 263–268 (not cited).

2. Stevens, N. T., Steiner, S. H. , & MacKay, R. J. (2018). Comparing heteroscedastic measurement systems with the probability of agreement. *Statistical Methods in Medical Research* 27, 3420–3435.

Extensions

1 Stevens *et al.* (2018) assume that $M_{ijk} \sim \mathrm{N}(0, \sigma_{ij}^2)$, with $\sigma_{ij}^2 = \sigma_j^2(s_i) = (\omega_j + \tau_j s_i)^2$.

Based on the replicates $Y_{ij1}, \dots, Y_{ijr}$, $r > 1$, the variances σ_{ij}^2 can be estimated and a maximum pseudo-likelihood solution might be proposed.
The replicates can be balanced (r) or unbalanced (r_{ij}).

Extensions

2 Model for several systems

$$Y_{i1k} = \qquad S_i + M_{i1k}, \tag{1}$$
$$Y_{ijk} = \alpha_j + \beta_j S_i + M_{ijk}, \tag{2}$$

for $k = 1, \ldots, r$ (replicates), $j = 2, \ldots, m$ (systems), and $i = 1, \ldots, n$ (subjects), noticing that $\alpha_1 = 0$ and $\beta_1 = 1$. $S_i \sim \mathrm{N}(\mu, \sigma_s^2)$ and errors M as in Stevens *et al.* (2017).

3 Marginal probability of agreement

$$\theta_j^{(M)} = P(|Y_{ij} - Y_{i1}| \leq c_j), \text{ for } j = 2, \ldots, m. \tag{3}$$

4 Conditional probability of agreement

$$\theta_j^{(C)} = P(|Y_{ij} - Y_{i1}| \leq c_j \mid Y_{il} = y_l : l \notin \{1, j\}), \tag{4}$$

for $j = 2, \ldots, m$.

5 Maximal probability of agreement
For $m = 2$ systems ($\alpha_2 = \alpha$, $\beta_2 = \beta$, and $c_2 = c$),

$$Y_{i1k} = S_i + M_{i1k}, \tag{5}$$

$$\psi(Y_{i2k}) = \alpha + \beta S_i + M_{i2k}, \tag{6}$$

for $k = 1, \ldots, r$ (replicates) and $i = 1, \ldots, n$ (subjects).

$$\theta^{(\max)}(s) = \sup_{\psi} P(|\psi(Y_{i2}) - Y_{i1}| \leq c \mid S_i = s). \tag{7}$$

$$\theta^{(\max)} = \sup_{\psi} P(|\psi(Y_{i2}) - Y_{i1}| \leq c). \tag{8}$$

Thank you!